Building models: deepening understanding by strengthening the links between mathematics and physics concepts

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12th May 2015, ROME

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Background

I have been interested in modelling since 2008

Initially based on VPython

Had little effect on conceptual understanding as measured by the FCI

Since 2011, concentrated on the process of modelling itself

The basic question:

Why model?

In physics, PER has shown the need for students to be active.

Then

How do we model?

How do we teach modeling?

How do we assess modeling and models?

Why model?

“The first question that arises frequently—sometimes innocently and sometimes not—is simply, "Why model?" Imagining a rhetorical (non-innocent) inquisitor, my favorite retort is, "You *are* a modeler." Anyone who ventures a projection, or imagines how a social dynamic — an epidemic, war, or migration—would unfold is running *some* model.”

Joshua M. Epstein (2008): Why Model? *Journal of Artificial Societies and Social Simulation* vol. 11, no. 4 12

This suggests that modelling is a natural part of thinking.

Echoed by several authors

- Nancy Nerssesian
- Ronald Giere

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"That humans (and animals) create internal representations of their environment (as well as of themselves) is probably the central notion in the cognitive sciences….Depending on the particular field within the cognitive sciences, one finds talk of such things as "schemata", "cognitive maps", "mental models" or "frames" " .

Giere, R, (1988) Explaining Science, Chicago, The University of Chicago Press

Nersessian (2008) :

MODEL BASED REASONING

The construction and manipulation of iconic, analogical mental models is considered as a form of reasoning in its own right.

Central conclusion:

Internal – external representations form a coupled system which facilitates model-based reasoning!

What is it about modeling that is so useful?

Epstein identifies 16 reasons for modelling, among which:

- Explain: Explanation is not the same as prediction
- Suggest dynamical analogies

  "It is a startling and wonderful fact that a huge variety of seemingly unrelated processes have formally identical models"

- Promote a scientific habit of mind

  militant ignorance—an iron commitment to "I don't know."

- Reveal the apparently simple to be complex and the complex to be simple

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What do we mean by a model?

My working definition (education):

- A representation

- constructed to explain or predict a physical phenomenon

- within a well defined theoretical framework (paradigm)

Model construction

David Hestenes

Modelling theory of physics education

For the most part, the *modeling theory* should appear obvious to physicists, since it is supposed to provide an explicit formulation of things they know very well. That does not mean that the theory is trivial or unnecessary. Much of the knowledge it explicates is so basic and well known to physicists that they take it for granted and fail to realize that it should be taught to students. A systematic explication of basic knowledge is an obvious prerequisite to the development of an instructional program which assures that the basics are adequately taught. When an instructor takes certain basics for granted and fails to teach them, the students flounder until they rediscover those basics for themselves or, more likely, develop inferior alternatives to cope with their difficulties. I submit that this unfortunate state of affairs is rampant in physics courses and contributes heavily to their legendary difficulty.

David Hestenes, Am. J. Phys, 55(5) 1987, 440-454

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Mathematical models in physics have 4 components:

A set of names for the objects and agents that interact with them

A set of descriptive variables representing properties of the objects

Equations of the model; structure and evolution

Interpretation

This is NOT a method, however.

We need to know how to get from the set of names and interactions to the mathematical expression

We need to understand the role of:

- knowledge;
- qualitative reasoning;
- spatial reasoning

Robert Glaser on representations

Robert Glaser, Education and Thinking: the role of knowledge, American Psychologist, 39(2) 1984 93-104

"We define a problem representation as a cognitive structure corresponding to a problem that is constructed by a solver on the basis of domain-related knowledge and its organization. At the initial stage of problem analysis the problem solver attempts to 'understand' the problem by constructing an initial problem representation".

The use of representations constitutes part of the thinking!

M Suwa and B Tversky:

"External representations contribute to the dynamic construction of ideas"

M Hegarty, B Meyer, H Narayanan, Diagrammatic Representation and Inference, Proc. Inference 2002 Lecture Notes in Artificial Intelligence series, Springer p341-343

- Free working memory
- Cue retrieval from long term memory
- Allow perceptual judgements about spatial relations
- Allow the generation of new ideas

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Glaser: the role of knowledge in thinking:

"... a major component of thinking is seen to be the possession of accessible and usable knowledge."

Robert Glaser, Education and Thinking: the role of knowledge, American Psychologist, 39(2) 1984 93-104

Implication: we need conceptual knowledge to be able to build models

The role of qualitative and quantitative knowledge

Hestenes : A set of;

- names for the objects and agents that interact with them;
- descriptive variables representing properties of the objects

Andrea di Sessa, (1987): Argues for the importance of qualitative knowledge: intelligent systems in AI have been constructed on the basis of qualitative knowledge.

> These systems can undertake very specific activities; "Such flow of control from qualitative dependencies into calculation seems quite general"
>
> "Novice physics students have been documented to display the kind of inefficient blind search through equations that … programmes avoid by using qualitative knowledge."

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Elements of a Theory of Modelling

- Structure of a model - Hestenes
- Domain-specific knowledge; qualitative precedes quantitative
  - agents and interactions, properties
- Spatial reasoning – Diagrammatic as well as visualization
  - understanding the problem
  - understanding the interactions
- Model-based reasoning – iconic analogical
- Mathematical knowledge – set down the equations
  - Quantitative reasoning
  - Translation of representations
- Application and evaluation: testing the model

A protocol for building models: ACME

- **Assess** the problem: what have we got? (qualitative)
- **Construct the Model**: work out the detailed interactions (quantitative)
- **Evaluate** the model: run it, look at the outcome, is it consistent?

Some models

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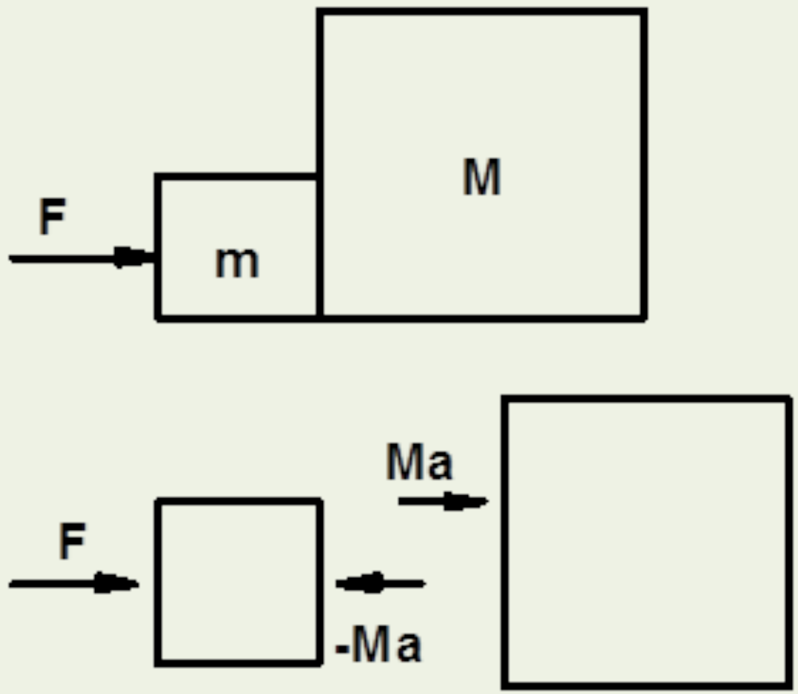
1. Newton's 3rd law

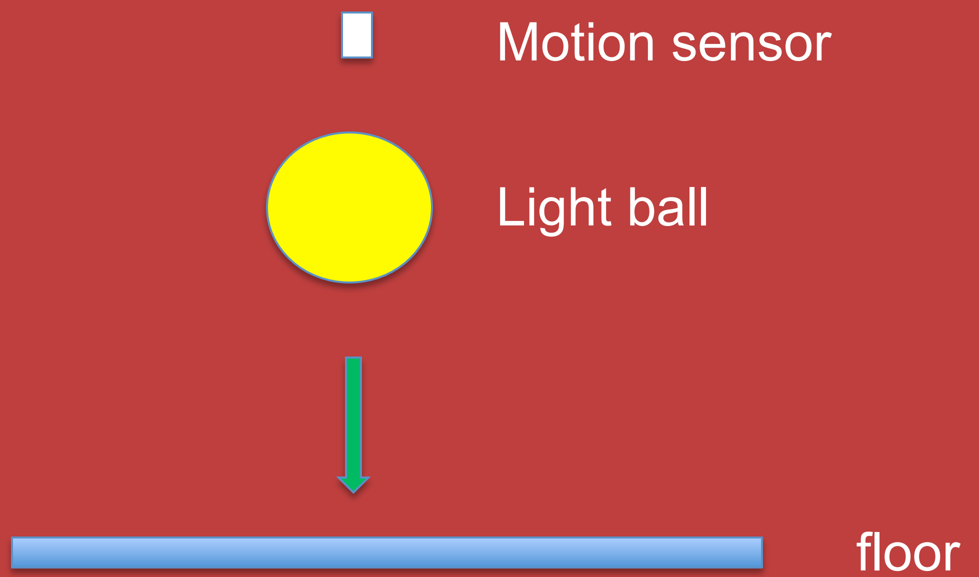
A force acts on a mass, m, which is accelerating. In turn it pushes a mass M. What are the forces acting on each?

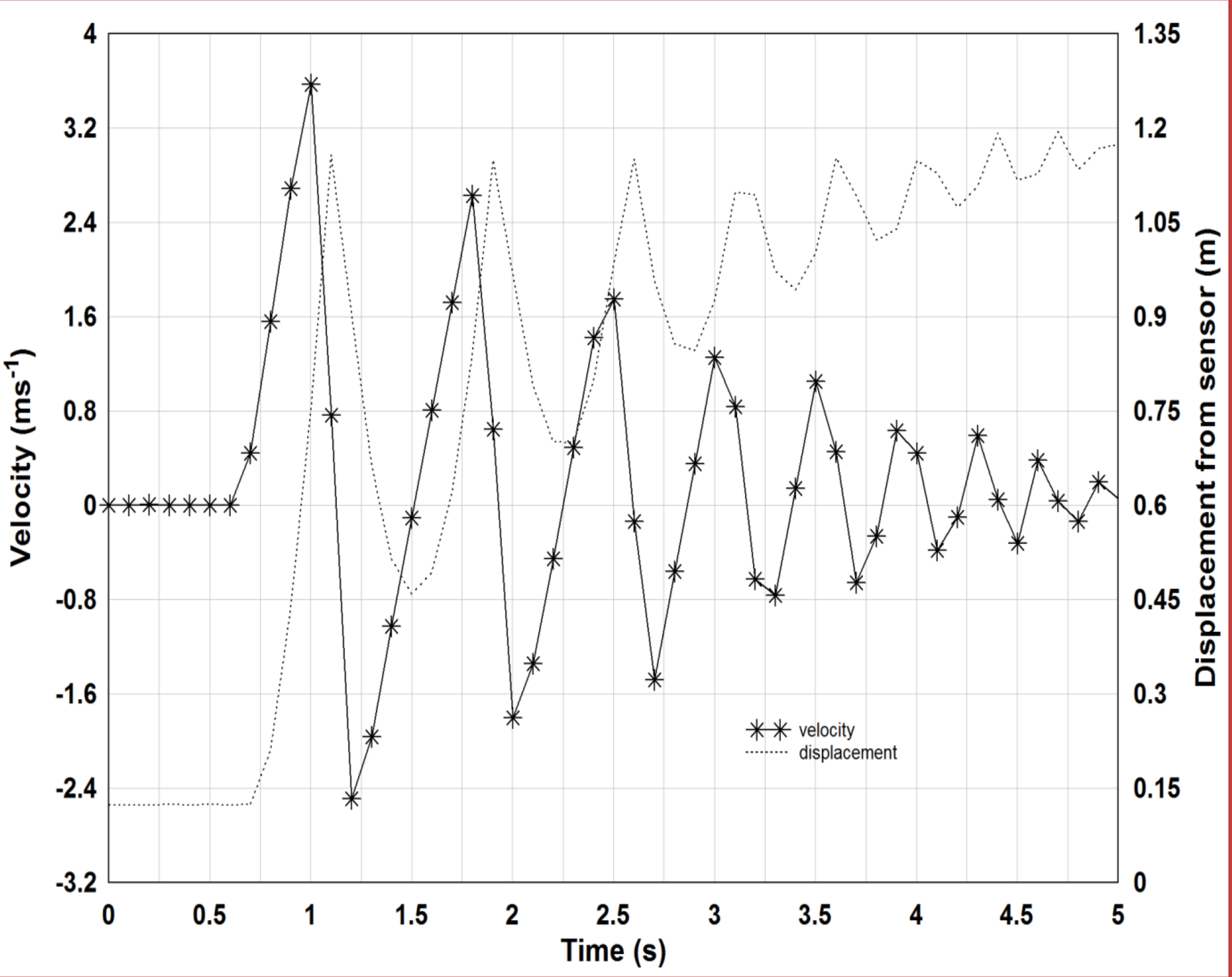


$F=(m+M)a$

$ma=F-Ma$

2. bouyancy





Velocity (ms-1)
Displacement from sensor (m)
Time (s)
velocity
displacement
4
3.2
2.4
1.6
0.8
0
-0.8
-1.6
-2.4
-3.2
1.35
1.2
1.05
0.9
0.75
0.6
0.45
0.3
0.15
0
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1.5
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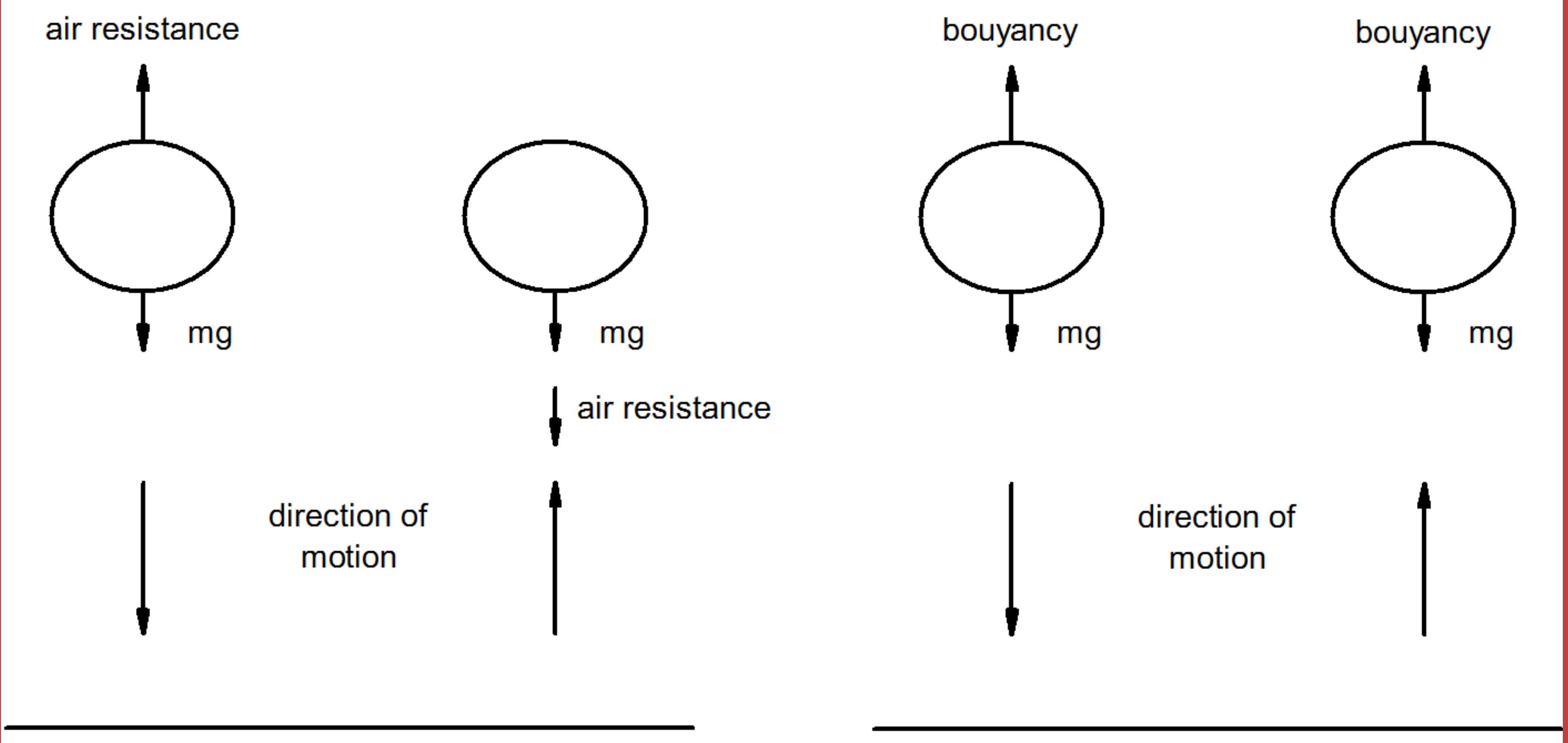
2.5
3
3.5
4
4.5
5


air resistance
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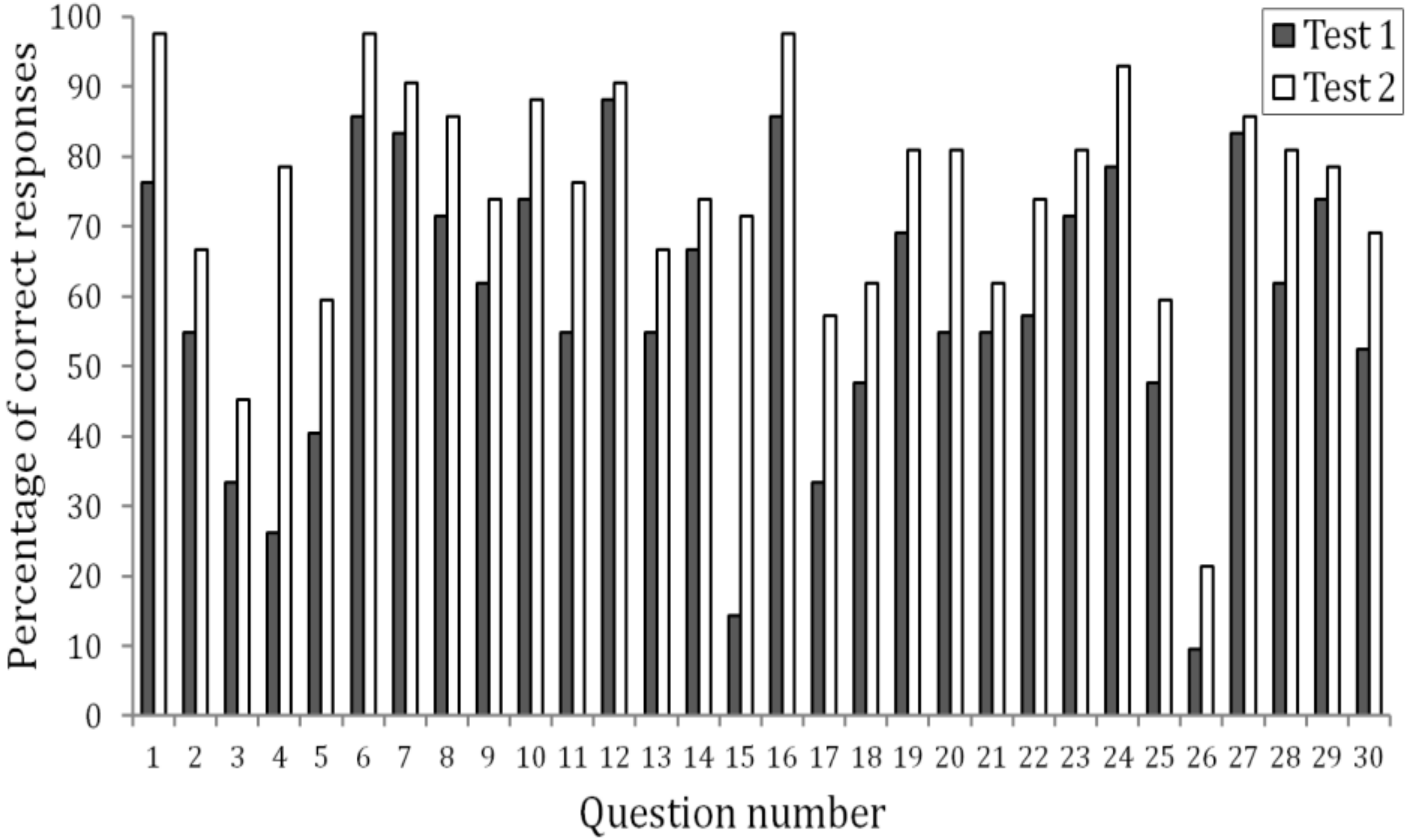
direction of motion



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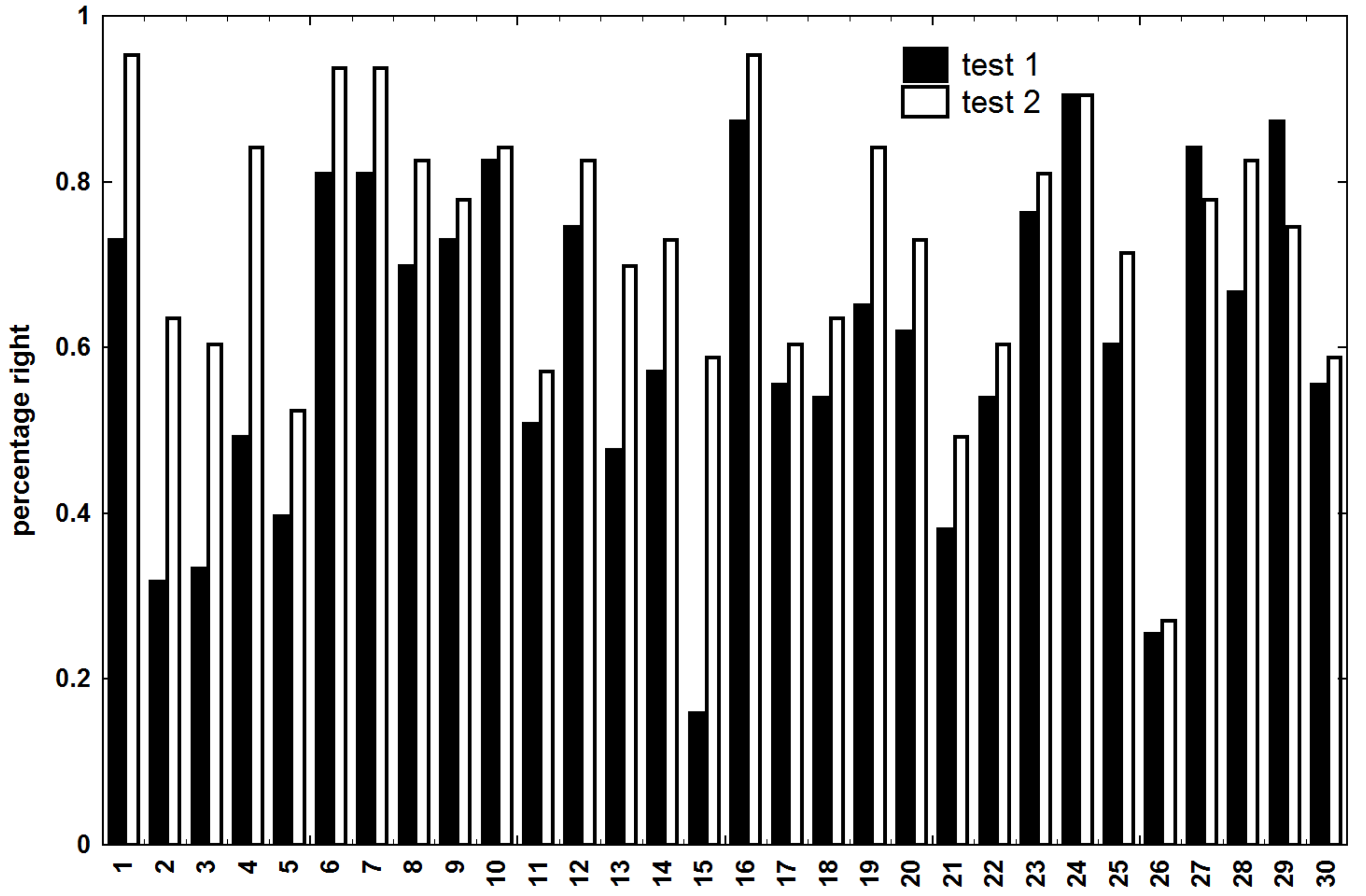
2010-11



percentage right
test 1
test 2
1
0.8
0.6
0.4
0.2
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

2011-12

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The lecturer was great at explaining & using examples to explain, was also good to be expanding on existing knowledge rather than everything being new.

The way Dr Sands makes us think - really helpful when doing tasks - thinks about Modelling, ACME etc. Really helps me think more about the problems that are presented to me.

Summary

- ❑ Constructing models promotes reasoning.
- ❑ student participation is essential, because there are several different and difficult aspects .
- ❑ They key to this approach lies in translating between representations.

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Fin.

Thank you
for your time.

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Assessing modelling

Assess the representations!

- Decide on the minimum number of representations for a model: minimum of 3, but more typically 4 or more
- 4-level marking scheme for each representation

representation	mark
Not there	0
Needs a lot of work	1
Nearly there	2
sufficient	3

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Pedagogical perspective: PER

"PER has come to see learning as an active process of engaging in directed cognitive activity to construct useful knowledge structures while practicing skills and mental processes."

(Gerace & Beatty, 2005)

"A functional understanding of physics connotes the ability to interpret and use knowledge in situations different from those in which it was initially acquired."

Lillian McDermott, Oersted Medal Lecture 2001:

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